The Pendulum

Goals and Introduction

In this experiment, we will examine the relationships between the period, frequency and length of a simple pendulum. The oscillation of a pendulum swinging back and forth is a familiar example of periodic, or harmonic, motion. If you imagine a child swinging back and forth on a swing, the child will repeatedly follow the same path, back and forth. As we describe the behavior and analysis of the pendulum, we will refer to this child-swing system to help us anchor our understanding.

In broad terms, a pendulum consists of an object (or “bob”), hanging from a pivot point, that is free to swing about the pivot point. To aid us in our analysis here, we will make a simplifying assumption for our pendulum – that all of the mass of our pendulum is concentrated at a point, which is tethered to the pivot point by a rope/cord/wire of negligible mass. When this assumption is made, the pendulum is referred to as a *simple pendulum*. Consider the child on the swing, again. If lightweight nylon ropes are used to attach the swing to the pivot point, this assumption isn’t too far from reality when the child sits on the swing!

The equilibrium position for the pendulum is when the cord is vertical, or when the bob is directly below the pivot point. When a child sits motionless on a swing, the child and swing are in equilibrium, as they will continue to remain motionless until the child is pushed or pulled to one side. Once displaced from the equilibrium position, the child will swing back and forth in simple harmonic motion, as long as the angular displacement is small ($\theta < 6°$). As seen in Figure 20.1, the pendulum bob maps out a portion of a circle as it swings back and forth. The radius of this circular path is equal to the length of the cord, L, connecting the bob to the pivot point.

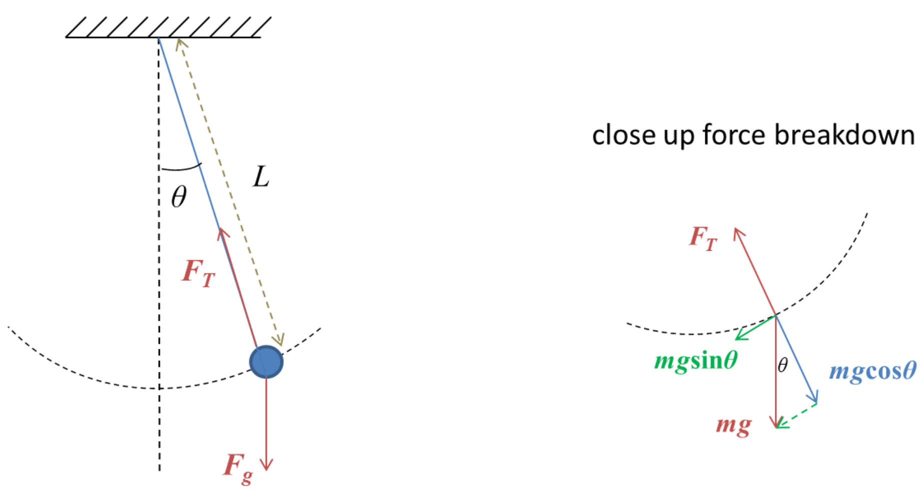

Figure 20.1

The angular displacement, θ, of the pendulum away from the equilibrium position is used to record the position of the bob as it oscillates, and the maximum angle the cord makes during an oscillation is called the *amplitude*. Imagine pulling the child on the swing until the swing made an angle of 5° with the vertical and then releasing the child. When you do this, the child swings back and forth, returning to a maximum angle of 5° every time he or she swings to the maximum forward and backward positions. Can you use conservation of energy to argue why this is so?

One complete oscillation of the pendulum could be described as moving from the maximum angle on one side, through the equilibrium position to the other extreme position, and then back through the equilibrium position to the original amplitude location. The dashed semicircle on the left side of Figure 20.1 indicates the full range of motion of the pendulum (note: the angles achieved in that figure are exaggerated to help us visualize the motion – the maximum angle should be less than 6° for simple harmonic motion). The time that is required to go through one complete oscillation is called the *period*, *T*. When the period is not dependent on the amplitude of the motion, we say that the oscillating behavior is an example of simple harmonic motion. An event that is periodic may also be described in terms of its frequency, *f*, or how many times the oscillation repeats per second. The period and frequency of an oscillation are related:

$$f = \frac{1}{T} \; . \qquad \text{(Eq. 1)}$$

Because the pendulum bob swings back and forth, changing speed and direction, there must be some acceleration responsible for this behavior. When you pull back the child on the swing and let go, the forces acting on the child-seat system are the gravitational force (always pointing straight downwards) and the tension in the cord (always pulling up towards the pivot point). We can better appreciate now why the vertical position is the equilibrium position. When the child is at the vertical position, the tension can point straight upwards and possibly be equal and opposite to the gravitational force. Once the swing is at some angle, however, there will be a component of the tension that acts to pull the swing towards the equilibrium position. When you pull the child back and let go, the cord pulls the child towards the vertical, thus accelerating the child. When the child reaches the vertical (equilibrium) position, there is no net force, but he or she is already moving and will continue moving. The child begins to slow after passing the vertical position because the cord is pulling the child towards the vertical position that he or she has just passed! This slows the child until he or she reaches the maximum angle on the other side and the speed begins to increase as the child is continually pulled towards the vertical position. In some sense, you could say that the child is always falling towards the vertical position; it's just that the existing motion causes the child to continually swing through it and oscillate back and forth.

A careful analysis of these forces leads to a prediction for the frequency of the simple pendulum when it is in simple harmonic motion.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad \text{(Eq. 2)}$$

Note that this relationship predicts that the frequency of the simple pendulum does NOT depend on the mass of the bob! Rather, it depends solely on the gravitational field (on Earth, 9.8 N/kg) and on the length of the pendulum, L. Realize, also, that the period, T, is dependent on these same two quantities, since the period and frequency are related by Equation 1.

Here, we will test the validity of Equation 2, and test to see if the mass of the bob affects the period, or frequency, of a pendulum. We will also measure the period for pendulums of different lengths in an attempt to verify the predicted value of the gravitational field on Earth, 9.8 N/kg.

Goals: (1) Measure and consider aspects of the simple pendulum.
(2) Make appropriate measurements and determine whether or not the mass of the bob affects the pendulum's frequency when it is in simple harmonic motion.
(3) Determine the gravitational field on Earth by measuring the period of several pendulums and compare to the accepted value.
(4) Based on the results, evaluate the validity of Equation 2.

Procedure

Equipment – string, mass holder with removable masses, meter stick (or other distance-measurement tool), balance, stopwatch, stand attached to table

1) The string should have one end attached to a support with enough clearance for it to swing freely. If this is not already done, attach the string to the support, as instructed by your TA, using a bow knot (the same kind of knot you use to tie your shoes).

> When you need to change the length of the string, you could wrap it around the arm of the stand where it is tied, keeping the mass holder attached to the end. Otherwise, you can keep the string tied on the arm and just tie the mass holder at different heights. The procedure is written using the latter method.

2) Using the balance, **measure and record** the mass of the mass holder. Label this as m_{mh}.

3) The length of your pendulum is the distance from the support point to the bottom of the mass holder. Attach the mass holder along the free end of the string, again, by using a bow knot, so that the length of the pendulum will be 1.25 m. **Measure and record** the length of your pendulum.

4) Either you or your lab partner should pull the pendulum back (keeping the string taut) so that the string makes a small angle with the vertical (about 5°). The other person should operate the stopwatch and reset it at this time. Practice using the stopwatch so that you know how to start, stop, and reset the watch, and understand how to read its results.

5) Recall that the period is the time to go through one oscillation. Here, that is the time it takes to return to your partner's hand. Rather than measure that time for one oscillation, measure the time for 10 oscillations. Practice the following a couple of times to become familiar with the process: The person holding the pendulum releases the bob. The person operating the stopwatch starts the watch when it returns to the initial position. Then begin to count the number of oscillations of the pendulum and stop the stopwatch when it completes its tenth cycle.

6) Pull the pendulum back so that it makes a small with the vertical and release it. **Measure and record** the total time for the pendulum to make 10 oscillations, t_{tot1}. Note that the current mass of the pendulum is just that of the mass holder. **Record** this mass as, m_1.

7) Place a 50-g mass on the mass holder. **Record** the new total mass of the pendulum as, m_2 (the 50-g mass plus the mass of the holder).

8) Pull the pendulum back so that it makes a small with the vertical and release it. **Measure and record** the total time for the pendulum to make 10 oscillations, t_{tot2}. Note that we have not changed the length of the pendulum. We can use these results to see if the mass affects the period, or frequency of the pendulum.

9) During the next set of measurements, the mass of the pendulum will not change. Currently, the mass holder has a 50-g mass. Add a 20-g mass to the mass holder. **Record** the total mass of the pendulum as, m_{tot} (the 20-g mass plus the 50-g mass plus the mass of the holder).

10) Tie the mass holder to the string so that the length of the pendulum (from the support to the bottom of the holder) is 1.25 m. **Measure and record** this length as, L_{A}.

11) Pull the pendulum back so that it makes a small with the vertical and release it. **Measure and record** the total time for the pendulum to make 10 oscillations, t_{totA_1}. Repeat this process two more times. **Record** your results as t_{totA_2} and t_{totA_3}.

12) Tie the mass holder to the string so that the length of the pendulum (from the support to the bottom of the holder) is 1.00 m. **Measure and record** this length as, L_{B}.

13) Pull the pendulum back so that it makes a small with the vertical and release it. **Measure and record** the total time for the pendulum to make 10 oscillations, t_{totB_1} . Repeat this process two more times. **Record** your results as t_{totB_2} and t_{totB_3} .

14) Tie the mass holder to the string so that the length of the pendulum (from the support to the bottom of the holder) is 0.75 m. **Measure and record** this length as, L_C.

15) Pull the pendulum back so that it makes a small with the vertical and release it. **Measure and record** the total time for the pendulum to make 10 oscillations, t_{totC_1} . Repeat this process two more times. **Record** your results as t_{totC_2} and t_{totC_3} .

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

In the first part of this experiment, you recorded two times for different masses on the same length of pendulum, t_{tot1} and t_{tot2} . Each of these represented the time for 10 oscillations. Using this fact, calculate the period for each case. Label your two periods as T_1 and T_2 respectively.

Calculate the frequency of the pendulum in each case, using Eq. 1. Label these as f_1 and f_2. Note that the frequency will have units of 1/s, often called a Hertz (Hz).

Question 1: Given your calculations here, does the frequency (or the period) of the pendulum depend on the mass of the bob? Do your results confirm the expectation? Explain.

In the second part of the experiment, you recorded time results for the oscillation of three different pendulums – each with the same mass but with different lengths.

Calculate the period for each total time in case A, labeling your results as T_{A1}, T_{A2}, and T_{A3}.

Calculate the mean period for your results in case A. Also, calculate the standard deviation and standard deviation of the mean for your three period measurements.

HINT: You may review the definitions of standard deviation and standard deviation of the mean, and how to calculate them, in the Statistical Analysis of Data lab.

Using Eq. 1 and the mean value of the period, calculate the frequency of the pendulum in case A. label this as f_A.

Repeat the calculations for the periods, mean period, standard deviation, standard deviation of the mean, and frequency for trials cases B and C.

Question 2: Given the prediction offered by Eq. 2, should the frequency increase or decrease as the length of the pendulum is decreased? Why? Did your data bear out this prediction? Explain.

Using Eq. 2, your calculated frequencies f_A, f_B, and f_C, and the recorded length in each case L_A, L_B, and L_C, calculate the value of g in each case. Label your results as g_A, g_B, and g_C.

Determine the mean of your results for g in each case and label this result as g_{avg}. Also, compute the standard deviation and standard deviation of the mean of your results for g.

Error Analysis

Question 3: Why did we measure the total time for 10 oscillations to eventually find the period of the pendulum, rather than just measure the time for one oscillation? Explain why this should result in a more accurate measurement of the period.

Given that the accepted value of g on Earth is 9.8 N/kg, compute the percent error between your measured/calculated value g_{avg}, and this accepted value.

Question 4: Remember that the standard deviation of the mean gives you a measure of the experimental uncertainty in your mean value. Thus, the results would say the true value lies between $g_{avg} \pm \sigma_m$. Considering your percent error, your mean value, and your standard deviation of the mean, did your experimental results confirm the accepted value of g on Earth? Explain.

Question 5: Considering your mean values and your standard deviations of the mean for the period in cases A, B, and C did your results confirm the expectation that the period will be different as the length of the pendulum is changed? Explain.

Questions and Conclusions

Be sure to address Questions 1-5 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "The Pendulum," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) Rachael and Ellie follow steps 1-6 of the procedure, measuring the mass of the empty mass holder to be 66 g and the length of their pendulum to be 98.2 cm. Predict what the frequency of their pendulum will be in Hertz *[Hint: convert to SI units before calculating]*.

PL-2) Ellie adds a 50 g mass to the mass holder. Predict the frequency of their pendulum in Hertz with the added mass. *[As before, the mass holder has a mass of 66 g and the length of the pendulum is 98.2 cm]*.

PL-3) Rachael lets out some string so the total length of the pendulum is now 135.4 cm. Predict the frequency of their pendulum in Hertz with the longer string.

PL-4) Rachael changes the length of the string again, and Ellie times the pendulum as it swings back and forth ten times, measuring 16.7 s. What is the frequency of their pendulum in Hertz?

PL-5) Ellie and Rachael measure the mean and standard deviation of the mean of the gravitational field strength to be $g_{\text{ave}} = 9.76 \pm 0.05$ N/kg. How does this compare to the accepted value of g?
(A) It is numerically equal to g.
(B) It is not numerically equal, but it is statistically equal to g, because of the standard deviation of the mean.
(C) It is neither numerically nor statistically equal to g, because of the standard deviation of the mean.
(D) We can't tell from the information given.